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ABSTRACT

This paper presents the results of a pilot study conducted to examine Web-based instructional content for mathematics. Two research questions were posed during the study: (1) how is technology being used to represent mathematical concepts online? and (2) how do the representations work together as a system? A mixed method content analysis design was used in this study to examine instructional content for five concepts ranging from basic to advanced mathematics. The results indicated a positive correlation between non-text representations and the total number of representations. Representational function mirrored that found in the taxonomy developed by Shaaron Ainsworth. (Contains 29 references.) (Author/KHR)

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Online Mathematics Instruction: An Analysis of Content

Chareen Snelson
Boise State University
csnelson@boisestate.edu

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Abstract

This paper presents the results of a pilot study conducted to examine Web-based instructional content for mathematics. Two research questions were posed during the study: (a) how is technology being used to represent mathematical concepts online? And (b) how do the representations work together as a system? A mixed method content analysis design was used in this study to examine instructional content for five concepts ranging from basic to advanced mathematics. The results indicated a positive correlation between non-text representations and the total number of representations. Representational function mirrored that found in the taxonomy developed by Shaaron Ainsworth.

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Online Mathematics Instruction: An Analysis of Content

Mathematics instruction through the Internet has moved from theoretical possibility to reality over the past few years. Online courses have become an increasingly available choice for students in K-12 and postsecondary institutions. A report presented to Congress noted rapid growth in online education with projected postsecondary enrollments expected to triple from 1998 to 2002 (The Web-Based Education Commission, 2000). Virtual schools for K-12 students have been planned or implemented in 14 states across the U.S. with calculus being cited as the most frequently offered online course (Clark, 2001).

Content for Online Mathematics Instruction

Instructional content for online mathematics courses may be obtained through one of three sources. An instructor may develop their own content, which is considerably time consuming (Allen, 2001). Content may be purchased through a vendor, which will save time but may involve substantial cost (Hirsch, 2001). Content may also be found on free public Internet sites, however, the quality of this content is variable (The Web-Based Education Commission, 2000). Free web-based content has the advantage of time and cost savings for instructors, yet little may be found in the research literature reporting systematic study of instructional content for mathematics available online. The focus of the present study is on the analysis of free Internet content for online mathematics instruction. This study serves to pilot a line of content analysis research that may be extended to include other sources of content for comparative purposes.

Communication Through Multiple Representations

Online mathematics instruction that is delivered asynchronously to remote sites is accessed through the computer interface. Mathematical ideas are communicated to the learner through one or more representations included in the instructional content. Traditionally, representations in mathematics have included verbal or sentential text, mathematical symbols, tables, and graphs or diagrams (Friedlander & Tabach, 2001; Goldin & Shteingold, 2001). New ways of representing mathematical ideas have emerged with the development of technology (Heid, 1997; Sonwalker, 2001). Technology based representations may include animations, audio and video clips, or interactive components that accept user input and provide automatic feedback. All of these are external representations that may exist within content presentation.

Burch (2001) articulated the concern that basic communication issues, including the user interface, have been a missing component in web-based distance education research. The importance of the user interface in asynchronous online mathematics instruction becomes elevated when considering that the learner is likely to access the majority of the instructional content through it. The research questions for the present study are, therefore, centered on the use of representations within the user interface. Two research questions were addressed in this study. The first question asked how technology was being used to represent concepts in online mathematics courseware. The second question asked how the representations work together as a system to present mathematical concepts.

Representations and Mathematical Communication

Representations are used to stand for something else, whether it is an idea or physical entity in the real world. Palmer (1978) developed the terms *representing world,* to refer to the representations and *represented world* to refer to what they stand for. The representing world maps onto corresponding features of the represented world. A textbook word problem featuring a volume calculation may use words to describe the dimensions of a swimming pool. Yet, a drawing possesses spatial properties that map more effectively to the spatial properties of the represented pool.

Designers of content for online mathematics instruction are faced with the task of deciding how to distribute information over one or more representations to communicate mathematical ideas. Choices are likely to be made based on the complexity of the mathematical concept and the properties of the representations. An algebraic equation such as $y = x^3 + 5$ may compactly represent the relationship between variables x and y. The variability of the solution set is made

salient by the spatial properties of the graph shown in Figure 1. Together the equation and graph form a system of representations that highlight two different aspects of a mathematical concept.

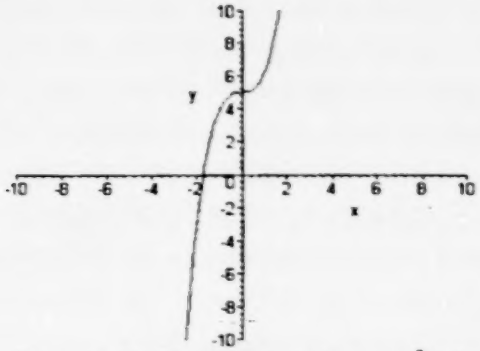

Figure 1: Graph of the equation $y = x^3 + 5$

The Role of Representations in Mathematics Education

Representation is the tenth standard in The National Council of Teachers of Mathematics (NCTM, 2000) *Principles and Standards for School Mathematics*. According to the NCTM standards, "The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas" (p. 67). For example, a division problem presented in Roman numeral notation would be more difficult to understand than one written in traditional Arabic base-10 form. Alternatively, a table can be used to align data to make patterns more recognizable.

Benefits of Multiple Representations

When students learn how to use a variety of representations they may be better equipped to solve a diverse array of problems. Training in problem representation skills has been associated with success in solving algebra word problems (Brenner et al., 1997). Schwarz and Hershkowitz (1999) have documented an increase in the number of examples students used in solutions to problems after working with computer software that enabled manipulation of multiple dynamic representations.

Individual differences in learners may be accommodated through multiple representations. The combination of different types of representations provides multiple entry points to concepts and corresponds to the various symbols systems favored by each of the intelligences described in multiple intelligence theory (Gardner, 1991). Cox and Brna (1994) theorized that learners may construct and use preferred representations based on differences in cognitive style. Differences in preference for visual, auditory, or tactile modalities may be accommodated by the inclusion of representations that allow the user to access information through a favored sensory mode (Guild & Garger, 1998; Shaughnessy, 1998). Visual content, audio clips, and interactive components in online content may accommodate these learning styles.

Learning Demands and Multiple Representations

A full understanding of mathematics requires an understanding of multiple representations, which have been referred to as the "...language of mathematics" (Coulombe & Berenson, 2001, p. 166). Fluency in mathematical representations includes an understanding of the *format* and *operators* of each type of representation. When reading the graph of a linear equation a learner would need to interpret the format of axes, scales, and labels. Additionally, if the learner uses the graph to extract information about a mathematical concept or real world application then they would need to understand the operations, or operators that may be used to access that information.

The understanding of format and operators are learning demands faced by students of mathematics. Translation between representations has also been cited as a learning demand particularly when the format of the representations differs considerably (Ainsworth, Bibby, & Wood, 2002). This presents a quandary for designers of instructional content. The differing formats of multiple representations allow multiple aspects of a knowledge domain to be illuminated. Yet, students may have difficulty understanding how to translate between the representations to take advantage of the combined properties of the multiple representational systems. A solution to this dilemma was proposed by Kaput (1989) who suggested the use of technology to dynamically link representations. For example, a table could be linked to a graph so that as values are entered into the table they automatically appear on the corresponding graph. This would provide a connection from one representation to another to support learner understanding of the relationship between them. Over time the support could be lifted allowing the technological representations to be used as a scaffolding mechanism.

Ainsworth, Wood, and O'Malley (1998) examined a variation of dynamic linking in a study of elementary students who were learning that multiplication problems can have more than one correct solution. Students worked with a software program that automatically produced both place value and tabular representations of problem solutions. These two representations displayed equivalent information in different forms. Results of this study indicated that the dynamic linking of representations facilitated greater understanding of multiple solutions presented in multiple forms.

Web-Based Content for Mathematics Instruction

Technology can be used to generate dynamic representations that are not available in traditional textbooks. Audio clips, video clips, animations, and interactive components with automatic feedback may be added to online content for mathematics instruction. The properties of these representations may serve to support learners in developing a deeper understanding of mathematical concepts. The extent to which technological representations are included in existing content for online mathematics instruction may be examined through content analysis methodology.

Content analysis is a research methodology described as "...a set of procedures to make valid inferences from text" (Weber, 1990). It is often used in social science research to examine communications content as symbolic phenomena (Krippendorff, 1980). Historically, the data for content analysis has been printed text such as newspapers or speeches. For example, propaganda research in World War II used content analysis methodology (Krippendorff, 1980). Now other types of communication media such as images, video, and audio recordings have found their way into content analysis research (Gall, Borg, & Gall, 1996).

In education, content analysis has been used to analyze textbooks and other forms of instructional media. Textbooks have been of interest due to their relationship to curriculum (Gall et al., 1996). Watanabe (2000) used content analysis methodology to examine and compare American and Japanese teacher's manuals for mathematics.

The development of technology has made it possible to create dynamic and interactive instructional content. This has led to the expansion of content analysis research to include electronic media (Gall et al., 1996). For example, Unsworth (1999) conducted research to compare the presentation of scientific content in traditional textbook format with that in computer CD format. Semiotic analysis was used to examine how the text and images in the two formats conveyed meaning about the concepts of floating and sinking. Unsworth (1999) declared a need for research into the use of electronic media for instruction when stating:

> Not only is this generally accepted as a research priority, but it is also agreed that the inter-relationships among the linguistic and the visual is in urgent need of research in view of the increasing prominence of multiple, interactive forms of semiosis via electronic multi-media materials. (p. 159)

Although Unsworth's study focused on science content, it seems reasonable to extend multi-media content analysis to other disciplines such as mathematics. In mathematics, electronic media can be used to deliver multiple representations of concepts. For the online student, this may facilitate learning in the absence of a face-to-face instructor.

Analyzing the Functions of Representations in Content

A taxonomy of the functions of representations was developed by Ainsworth (1999) to frame analysis of how representations work together in systems. One of the proposed uses of the taxonomy is to examine existing content. The functions of multiple representations may be complementary, constraining, constructing, or a combination of functions. Complementary representations each provide information or support different processes. A graph complements an algebraic equation by contributing information about variation in the solution set. Constraining representations are used to prevent misunderstanding through familiarity or inherent properties. A table can be used to align data in order to prevent misinterpretation of membership within data categories. Constructing representations support extension to other knowledge domains or an increase in abstraction. The animation of a ball traveling through parabolic motion can be used to extend quadratic equation concepts to physics.

The analysis of representations in online instructional content can include the taxonomy of functions to examine how each representation contributes to the communication of a mathematical idea. This analysis may be combined with a quantitative analysis of frequency counts that will document the extent to which each type of representation is used. The combination of qualitative and quantitative methods will produce an enhanced view of the current status of representation use in content designed for online mathematics instruction.

Methodology

A mixed-method content analysis research design was used in this pilot study of content developed for online mathematics instruction. According to Weber (1990), "The best content-analytic studies use both qualitative and quantitative operations on text. Thus content analysis methods combine what are usually thought to be antithetical modes of analysis" (p. 10). This pilot study included both quantitative and qualitative analysis of online content for mathematics instruction.

The unit of analysis was the portion of a free public web site that contained instructional content for the five mathematical concepts shown in Table 1. This provided a cross section of content ranging from basic to advance mathematics.

Table 1
Mathematical Concepts Chosen for Analysis

Mathematics Domain	Mathematical Concept
Basic Mathematics	Multiplication (Integers)
Algebra	Quadratic Equations
Geometry	Quadrilaterals
Statistics	Normal Distributions
Calculus	Integration by Parts

The dynamic and unstructured nature of the Internet precludes random sampling of specific types of content. Therefore, a purposeful sample of content was obtained through the use of searching software. This software, developed for online research, was used to simultaneously search 33 Internet search engines during the month of February 2002. Ten sites were randomly selected from each concept area for a total of 50 sites. Patton (1990) described this process as purposeful random sampling.

Quantitative Content Analysis

Quantitative methods in content analysis often involves the generation of categories and frequency counts of content falling within those categories (Gall et al., 1996; Krippendorff, 1980; Weber, 1990). In this study frequency counts of eight categories of representations were generated and used in a correlational analysis to examine relationships between representations (see Table 2).

Table 2
Representation Categories for Quantitative Content Analysis

Categories	Descriptions
Mathematical Text	Mathematical symbols used to convey mathematical concepts.
Verbal Text	Non-mathematical symbols used in words, sentences and lists.
Graphics	Graphs, diagrams, or images related to the concept.
Tables	Tabular organization of content.
Animations	Instances of graphics or text with motion.
Audio	Stand-alone audio clips.
Video	Video clips of instructional content such as teacher presentations.
Interactive Components	Interactive practice problems or graphics that are affected by user input.

The Kolmogorov-Smirnov and Shapiro-Wilks tests were run to determine normality of the distributions of data. Significant results on these tests ($p < .05$) indicated that the distribution of the data was significantly different from a normal distribution and non-parametric. Normality plots and scatter plots were generated to visually inspect the data.

Spearman's Rho and Kendall's Tau were used in a correlational analysis of the data. Kendall's Tau has been reported as being superior to Spearman's Rho in handling tied ranks and was included to provide additional evidence of significant correlations (Field, 2000).

Qualitative Content Analysis

The qualitative component of this pilot study was used to analyze how multiple representations of mathematics concepts work together as a holistic system. According to Patton (1990) the purpose of a systems study is to ask "How and why does this system as a whole function as it does?" (p. 78). Each online mathematics site was treated as a system to determine how the representations work together to present concepts. The systems of representations were analyzed using a qualitative inductive process that allowed themes to emerge. Emergent themes were compared to the taxonomy of representations developed by Ainsworth (1999) during data analysis. The qualitative data was triangulated with the quantitative data in a multiple method process (Patton, 1990). The combination of quantitative and qualitative data was used to produce a more coherent analysis of how representations are currently used in content for online mathematics instruction.

The instructional content was coded and analyzed by a single individual, which may be considered a limitation. Often, content analysis studies involve teams who code content and calculate inter-rater reliability. In this study, the single coder method provided a type of reliability that has been referred to as stability in content analysis texts (Krippendorff, 1980). Coding categories were also clearly defined and easy to differentiate when coding. This helped to minimize the limitations of single coder methodology.

Results

Frequency counts of eight categories of concept representation were generated during the quantitative content analysis portion of the pilot study. These categories included verbal text,

mathematical text, graphics, tables, animations, audio clips, video clips, and interactive components.

The number of different types of representations appearing in the instructional content was counted. For example, if the content contained verbal text, mathematical text and graphics it received a frequency count of three for the number of representations used. Within the fifty instructional units 30% used two representations, 40% used three representations, 14% used four representations and 16% used five representations. Technological representations (animation, audio clip, video clip, or interactive component) were included in 30% of the cases with the majority of those cases (26% of total) containing only one technological representation. In 4% of the cases two technological representations (animations and interactive components) were included in the instructional content. Video clips were not found in any of the cases.

Correlational Analysis

A correlational analysis was conducted to examine whether or not representations covaried across the aggregated data. A 2-tailed correlation was run on the seven existing representation categories and the total number of representations found per unit. Significant correlations are shown in Table 3.

Table 3
Significant Correlations

Correlation Between:	Spearman's Rho	Kendall's Tau
Mathematical Text and Tables	0.357*	0.276*
Verbal Text and Tables	0.321*	0.247*
Mathematical Text and Verbal Text	0.569**	0.399**
Verbal Text and Graphics	0.389**	0.302**
Audio Clips and Total Number of Representations	0.341*	0.313*
Tables and Total Number of Representations	0.605**	0.535**
Graphics and Total Number of Representations	0.694**	0.582**
Animations and Total Number of Representations	0.386**	0.351**
Interactive Components and Total Number of Representations	0.411**	0.361**

*$p < .05$, **$p < .01$

Representation Categories and the Ainsworth Taxonomy

The qualitative analysis focused on examining the functions of the representations within the instructional system. The seven types of representations found within the sample were analyzed in relation to how their observed functions fit into the Ainsworth (1999) taxonomy. Observed functions are shown in Table 4.

Table 4
Observed Functions of Representations

Representation	Function		
	Complement	Constrain	Construct
Verbal Text	X	X	X
Mathematical Text	X		X
Tables	X	X	
Graphics	X	X	X
Animations	X	X	
Interactive Components	X	X	
Audio Clips	X		

The complimentary function was the most commonly observed function. This function was typically used by two or three representations to provide supporting information about a concept. It also provided multiple perspectives, or multiple entry points to a concept. An example of this in the normal distributions content occurred when mathematical text, verbal text, and a graphic were used together to demonstrate the concept of the statistical mean. The mathematical text showed the data and the steps in the calculation of the mean. The verbal text described the process and the graphic illustrated where the mean was located within the distribution. Each representation complemented the other and provided a richer presentation of the concept.

The constraining function was used to help prevent misunderstandings from occurring. Tables were used in this function to organize information into a format that was easier to read and interpret. Tables were also used to clarify and organize steps in the tabular method of integration by parts. Verbal text was essential in constraining misunderstanding when interpreting mathematical symbols or explaining procedures. There were instances where mathematical derivations were presented without verbal text support. These presentations seemed more likely to lead to misunderstanding when faced by a learner who is new to the concept.

The constructing function was used in the sample to provide extensions to concepts. The primary method was to use applied examples of a concept. In the quadratic equations content the constructing function was used to show how quadratics sometimes appear in everyday life. A picture showing a suspension bridge was used along with verbal text and a graphic of the quadratic equation to support this idea.

There were occasions where representations assumed more than one function. Verbal text was often used to complement and constrain. Graphics were also used to complement and constrain when used to illustrate the normal distribution and the areas of division of standard deviations. There were no instances when a representation could not be categorized into one or more of the taxonomy categories.

Discussion

Two research questions were explored in this pilot study. The first question asked how technology was being used to represent concepts in online mathematics courseware. The second question asked how the representations work together as a system to present mathematical concepts. These questions were answered through quantitative and qualitative content analysis research methodologies.

The number of representations used in the instructional content ranged from two representations to five representations. Only two or three representations were used in 70% of the cases. None of the units of instructional content used all of the categories of representation and video clips were not included in any of the sampled content.

All of the units of instructional content were in electronic form accessible by computer. This form allows representations to be used that are not possible in a conventional textbook. For example, a printed textbook can have verbal text, mathematical text, graphics, and tables, but cannot support animations, audio clips, video clips, or interactive components. These require some type of electronic medium. Technological representations appeared in only 30% of the sampled content despite the technological capabilities available to support them. The majority of the content examined in this study was developed with traditional representations in static form.

A correlational analysis of the aggregated data indicated a positive relationship between non-text representations and the number of representation categories found in the instructional content. It could be concluded that instructional content is built from a core of traditional representations up to a multiple representation environment that incorporates non-text and technological representations. This is logical considering that technological representations such as animations require other representations to act on such as text or graphics.

Systems of Representations

The second question posed in this pilot study asked how the representations work together as a system to present mathematical concepts. Each category of representation was examined qualitatively to determine how it functioned within the instructional system. The functional taxonomy of representations developed by Ainsworth (1999) was used to categorize these functions. The taxonomy provides three main classifications of function. These functions are complimentary, constraining, and constructing.

Ainsworth (1999) proposed that contradictory results reported in multiple representation research may be explained by an examination of the functions of representations. The design of instructional content can be improved by carefully combining representations to take advantage of the properties and functions of each one. A poor fit between representations can lead to learner difficulties as attempts are made to translate between different representations. A multiple representation environment is only as good as its design.

The representations used in the pilot study sample seemed to fit well together in the majority of the cases. The presence of verbal text seemed to be the critical representation throughout the content. It often became the interpreter of other representations in lieu of a teacher who verbally explains concepts to students. Verbal text worked like mortar holding the rest of the representational bricks together.

Implications

This study has provided a snapshot of instructional content for mathematics that may be found on the public Web. The Internet is a dynamic place and future studies may show that the content used for mathematics instructions has evolved. Projects designed to provide open access to instructional content might impact the quality of free resources.

The rapid growth of online education including mathematics makes the study of content even more essential. Online instructors grapple with issues related to content. Content development is time intensive and some instructors have turned to either free Web content or commercial courseware to provide materials that will support their course and provide a quality learning environment. This pilot study has been undertaken as a first step in exploring how educational theory and content analysis methodology can be used in the evaluation of Web-based instructional content for mathematics.

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